

**Speculative Equilibria of  
"Managed" Primary Commodity Markets**

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### **Abstract**

This paper sets up a model of commodity speculation in the presence of a public buffer stock intervention. Equilibrium is characterized, existence is proven and sufficient conditions for uniqueness are given. These take the form of the buffer stock having "marginal propensities to store" between zero and one. A limited policy neutrality result is proven, but where the buffer stock does have an effect, some strong contrasts with the laissez-faire case are established. For example, the equilibrium is not necessarily a constrained social optimum; a sufficient condition is given under which it is. Further, examples are given in which a putatively stabilizing buffer stock intervention actually creates a price bubble where otherwise none could exist.

## 1. Introduction.

Primary commodity markets are terribly volatile. To many countries which depend on exports of a small number of primary commodities, this volatility is a terribly important thing, keeping them farther from their goals on debt, government revenues, investment, and everything else relevant to economic policy. As a result, stabilization policies have often been proposed in international discussions, and some, most often buffer stock schemes, have been implemented<sup>1</sup>. Yet we have no general theory of the effects of buffer stocks.

This paper attempts to begin one. Its goal is to find basic properties of market equilibrium under a buffer stock equilibrium: existence conditions, when equilibrium is unique and when it is not, and basic welfare properties. An attempt is made to provide these for a wide class of buffer stock rules, rather than simply those that have already been tried, since this is a clear prerequisite to any discussion at all of *optimal* buffer stock rules.

The model offered here is a generalization of some standard models in the literature. It assumes perfect competition. It is naive in that it does not allow for asymmetric information, the focus of some other competitive theories of commodity markets (Grossman, 1977; Stein, 1987), or transactions costs, the focus of some others (Williams, 1987). As a first pass, we abandon some of the richness of these other models. We do, however, note where possible how the results change when imperfect competition is allowed.

Large strides have been taken in recent years in the theory of competitive primary commodity markets. The basic case for intervention was scrutinized by Newbery and Stiglitz

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<sup>1</sup>See, for example, Gordon-Ashworth (1984) for a history.

(1981) in a penetrating study. They dealt with the question of optimal buffer stock interventions, but following Gustafson (1958), treated *all* storage as if it were under the planner's control, rather than dealing with the more realistic case in which a sector of competitive speculators is beyond the direct control of the authorities<sup>2</sup>. Recent advances in dynamic equilibrium theory have allowed Scheinkman and Schectman (1983) to develop quite rich analytical results for speculative equilibrium under perfect competition, and Deaton and Laroque (1992) to calibrate well-specified, fully dynamic numerical models of primary commodity markets, with suggestive results. However, these studies assume *laissez-faire*, while most of these markets have been subject to heavy and very frequent intervention. Salant (1983) explores the consequences of imposing a particular buffer stock rule on a primary commodity market, and Wright and Williams (1982, 1988, 1991 (Ch. 13, 14)) have explored the consequences of a small number of other rules, through simulations, assuming particular parameter values and functional forms for demand. These four studies take full account of the behaviour of speculators, but allow for a very limited class of policies or none at all.

Thus, what is needed is not found in the literature: a model which can both (i) allow for a wide class of buffer stock rules and (ii) deal with the behaviour of independent speculators in response to that rule. This paper is intended to fill that gap.

We find two key contrasts with previous results on unmanaged markets. First, the behaviour of risk neutral speculators is not necessarily socially optimal given the buffer stock. This is because the presence of the buffer stock sets up a kind of artificial externality. This

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<sup>2</sup>Strictly speaking, this applies to their Chapter 30. In Chapter 29, the approach is different; they consider only public-sector storage schemes under which there would never be an incentive for private storage. See Newbery and Stiglitz (1981, p. 414).

argument seems to be new<sup>3</sup>; previous examples of rational, socially inefficient speculation have relied on private information (Stein, 1987), which is absent here. Second, it is possible that the presence of the buffer stock can create multiple equilibria in a market which would otherwise have only one equilibrium. In a precise sense, then, "management" of the market may lead to the existence of bubbles, or socially inefficient second equilibria even when there is an efficient one. We offer examples and show conditions which rule this out.

Section 2 lays out the model; Section 3 studies its welfare properties, which turn out to be very different from those of the laissez-faire models; Section 4 develops existence and weak uniqueness results for equilibrium; Section 5 displays cases of multiple equilibria; Section 6 summarizes.

## 2. Basics.

We consider a market for a primary good. Consumers of this good take its price each period,  $p_t$ , as given and demand a quantity  $Q_t$  according to a stable inverse demand curve,  $p_t = P(Q_t)$ , with  $P(\cdot)$  continuous, bounded and strictly decreasing. We could easily take this as giving the demand for the raw good on the part of competitive processors, with  $p_t$  equal to the consumer price minus marginal processing costs.

There is a harvest  $h_t$  each period, with  $h_t$  an identically and independently distributed

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<sup>3</sup>However, an analogous argument has been made in the theory of investment, as will be noted later. Williams and Wright (1991, p. 389) find socially excessive rational private storage in their computer simulations of a model of a price floor buffer stock, but do not have a theoretical argument to explain it.

random variable. We could easily let harvests be serially dependent, and could easily allow for noisy signals of future harvests, without materially changing the results. For example, we could let the harvest and some signal  $\xi_t$  follow a joint  $M$ -order Markov process. In that case, matters would be slightly complicated by the need to keep track of the joint history of  $h_t$  and  $\xi_t$  going back  $M$  periods, which we denote  $H_t$ . Where these details would make a difference will be noted as we proceed.

Between the passive producers on the one hand and the consumers on the other we interpose a sector of risk neutral speculators, who buy the good and store it to earn capital gains. They are perfectly competitive; they take the price at any moment as given and make zero profits in equilibrium. There are no fixed costs to storage, but there is a per-unit per-period cost of  $k$ , and there may be an aggregate capacity constraint  $K$ . (If not, we will feel free to say that " $K=\infty$ ".) A fixed proportion  $\delta \geq 0$  of goods stored depreciates each period, and the interest rate is such that speculators discount future revenues at the rate  $\beta$ .

Onto this structure we plant an authority which has been created to manage prices in the market by running a buffer stock. This authority follows a rule<sup>4</sup> whereby it carries a volume of

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<sup>4</sup>Previous literature in this area has for the most part represented buffer stock rules by letting stocks carried by the authority be a function of the current price (for example, Wright and Williams (1988), Salant (1983)). This is natural at first blush since this is the way such policies are usually described in the real world. However, for analytical purposes it seems rather confusing, since the point of intervention is to *determine* the current price, which seems incompatible with *conditioning* the policy *on* the current price. More importantly, conditioning policy on price typically results in conspicuous indeterminacies. For example, if the authority follows a price band rule, then the price will often be constant over a wide range of values of the state vector, while the authority constantly adjusts its stocks up or down to keep it from bursting out of the band. If policy is a function of the price, though, how can we rationalize the constant changes in its holdings over that range while price is constant? Salant (1983) steers around this by referring to policy as a "correspondence" of price, rather than a function, but this is clearly not right, since in a period when the price is jammed up against an endpoint of the

stocks equal to  $\gamma(x_t, x_t^a)$  out of the period, where  $x_t$  equals total inventories (public and private) brought into the period plus  $h_t$ , i.e., total availability at time  $t$ ;  $x_t^a$  stands for public sector stocks brought into the period; and  $\gamma$  is continuous. One way of thinking of  $\gamma$  is that it could be the policy rule which results from the maximization of some objective function of price; for example, the expected length of time before the price breaks out of some assigned band<sup>5</sup>. Of course,  $\gamma$  must satisfy<sup>6</sup>  $0 \leq \gamma(x_t, x_t^a) \leq x_t$  for all  $x_t$  and  $x_t^a$ .

Thus the two variables describing the state of the market at any moment are  $x, x^a$ . We will often denote the vector concatenating these as " $s$ ". If we allowed for serially dependent harvests or signals of future production as described above, the relevant history  $H_t$  of the harvests and signals would need to be added to the state vector.

Equilibrium requires that if speculative stocks carried out of the period are less than  $K$ , then the expected profit per period not be positive (or speculative demand would be  $K$ ), and if speculative stocks carried out of the period are strictly positive, the expected profit per unit must not be negative (or speculative demand would be zero). Thus, denoting speculative stocks carried out of period  $t$  as  $J_t$ , equilibrium requires that:

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band the authority is by no means *indifferent* between different levels of stock holding, but must adjust stocks in each period just enough to compensate for changes in supply. The only way to describe the policy precisely is to indicate what the authority will do for any given value of the state vector, and that is why that approach is followed here.

<sup>5</sup>It does not matter for our purposes whether such maximization was conducted with or without benefit of commitment. Optimal plans and time-consistent plans (see Kydland and Prescott, 1977) would both be representable as a function of the current state of the market.

<sup>6</sup>It is also likely that the authority has its own capacity constraint, in which case  $\gamma$  will be bounded above, and that this bound will be well below  $K$ .



$$\begin{aligned}
 (2.1) \quad & J_t < K \Rightarrow p_t \geq \beta(1-\delta)E_t p_{t+1} - k \\
 & J_t > 0 \Rightarrow p_t \leq \beta(1-\delta)E_t p_{t+1} - k
 \end{aligned}$$

at all times. In addition, speculative storage must be feasible, meaning that it can never be negative or exceed  $K$  or the total amount available for purchase. Thus,  $J_t \in [0, K] \cap [0, x_t - \gamma(s_t)]$ . We limit our inquiry to "stationary rational expectations equilibria" (e.g., Deaton and Laroque (1992)), i.e., equilibria taking the form of a continuous function  $\psi$  of the state vector such that if  $p_t = \psi(s_t) = \psi(x_t, x_t^a)$  for all  $t$ , then (2.1) will hold for all  $t$  with certainty. From here on in, we will simply call this an "equilibrium".

Let the domain of  $\psi$  be  $D \subseteq \mathbb{R}^2$ . The largest set  $D$  could be is  $D_{\max} = \{(x, x^a) \mid 0 \leq x^a \leq x\}$ . It will generally be possible, and, often (as we shall see) desirable to let the domain be a proper subset of  $D_{\max}$ . In this case, we clearly will need to insist that equilibrium behaviour never give rise to a state vector outside of  $D$ ; thus we require that  $s \in D$  implies that  $(1-\delta)(x - P^{-1}(\psi(s))) + h' \in D$  almost surely. We might call this property "closure under  $\psi$ ". If  $D \subset D_{\max}$ , we might call  $\psi$  a "restricted" equilibrium.

Under our assumptions, total stocks carried out of period  $t$  will be  $x_t - P^{-1}(p_t)$ , so total availability in period  $t+1$  will be  $(1-\delta)(x_t - P^{-1}(p_t)) + h_{t+1}$ . This and the conjugation of different cases in (2.1) lead to the conclusion that an equilibrium  $\psi$  on  $D$  must satisfy:

$$\begin{aligned}
 (2.2) \quad & \psi(x, x^a) = \text{middle} \{ P(x - \gamma(x, x^a)), \\
 & P(x - \gamma(x, x^a) - K), \\
 & \beta(1-\delta)E_{h'}[\psi((1-\delta)(x - P^{-1}(\psi(s))) + h', (1-\delta)\gamma(s))] \}
 \end{aligned}$$

at all points in  $D$ , where  $\text{middle}(a,b,c)$  equals the middle value of  $a$ ,  $b$  and  $c$ <sup>7</sup>, and  $h'$  denotes the next period harvest. This is illustrated in Figure 1, where the heavy curve represents  $\psi$  for a given value of  $x^a$ .

The meaning of (2.2) is that there are three conditions in which the market may find itself at any moment. There may be a "stock-out," in which speculators sell all of the stocks they own because of expected losses on them, at which time the price will be  $P(x-\gamma(x,x^a,h))$ . There may be a period in which speculators are "stocked-up," or store to capacity because of positive expected profits on each unit stored, at which time the price will be  $P(x-\gamma(x,x^a,h)-K)$ . Finally, there may be "normal speculation", at which time traders hold some stocks but not up to capacity, and the price will be the discounted expected price next period. We will refer to values of the state vector giving rise to each of these three conditions as "stock-out points," "stocked-up points," and "normal speculative points" respectively.

Thus, any function mapping  $D$  into the range of  $P$  and satisfying the "closure" property on  $D$  is an equilibrium if and only if it satisfies equation (2.2)<sup>8</sup>. Having defined equilibrium in the managed market, we offer some discussion of its properties.

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<sup>7</sup>In the event that " $K=\infty$ " or that  $K > x-\gamma(x,x^a)$ , set  $P(x-\gamma(x,x^a)-K)$  equal to some very high number. Then the value of  $\psi$  will simply be the maximum of the other two terms.

<sup>8</sup>With signals and serially dependent harvests, all functions would be functions of the history  $H_t$ , and all expectations would be conditional on  $H_t$  and would be taken over next year's signal  $\xi'$  as well as the harvest.

### 3. Welfare properties: the collapse of constrained efficiency.

Here we ask the question: is the equilibrium a socially optimal allocation of resources taking the buffer stock policy as given?

If there were no buffer stock, private speculation in this market would maximize the social surplus (in a sense to be made precise below). This was pointed out by Gustafson (1958) and Samuelson (1971). Thus, adding a buffer stock can only make things worse; it is a distortion imposed on a previously undistorted market. This does not mean that it is undesirable, since the more thoughtful arguments for commodity price stabilization usually cast it as a second-best response to a distortion in some other market, for example, a missing insurance market (Newbery and Stiglitz, 1981, Ch. 15). However, the *extent* of the deadweight loss they create in the commodity market itself is clearly an issue. Part of assessing that involves asking whether private storage attenuates or exacerbates the distortion imposed by the public storage. That is the question at hand in this section.

Assume for the moment that  $k=0$ . Then farmers in period  $t$  receive  $p_t h_t$ , speculators receive  $p_t$  times the reduction in their stocks during period  $t$ , and consumers can be thought of as receiving  $U(Q_t) - p_t Q_t$ , where  $U(Q) = \int_{q=0}^Q P(q) dq$  and  $Q_t$  is the amount consumed in period  $t$ . The sum of these three is given by  $U(Q_t)$ , and thus  $U$  measures the sum of consumer and producer surplus. In the absence of a buffer stock, private storage maximizes  $V = E_t[\sum_{i=0}^{\infty} \beta^i U(Q_{t+i})]$ , where the expectation is conditional on all information available at time  $t$ ; this can be seen from the first order conditions for the maximization problem (Samuelson, 1971). It should thus be expected that  $V$  will not ordinarily be maximized by equilibrium in the presence

of a buffer stock, but does private storage maximize welfare *subject to the buffer stock rule as a constraint along with the harvest and storage technologies*? If it does, we will call it constrained efficient.

Formally, let  $\psi$  be an equilibrium on  $D$ , and let  $V$  be a function on  $D$  which satisfies the Bellman equation:

$$(3.1) \quad V(s) = \max_{\substack{J \in [0, K] \\ J \leq x - \gamma(s)}} \left\{ U(x - J - \gamma(s)) + \beta E_h [V((1 - \delta)J + \gamma(s)) + h', (1 - \delta)\gamma(s)] \right\}$$

Thus,  $V$  is maximized welfare given the constraints that private storage  $J$  never exceed private capacity or grain left over from public storage. If  $J(s) = x - P^{-1}(\psi(s)) - \gamma(s)$  gives a value for  $J$  which maximizes the right hand side of this equation for any value of  $s$ , then we will say that  $\psi$  is constrained efficient.

Managed equilibria sometimes can be constrained efficient. A trivial example is when the buffer stock has no effect on equilibrium; see Theorem 5. Here is a more interesting sufficient condition:

**Theorem 1.** Suppose that  $\gamma$  depends only on total availability  $x$ , and is non-decreasing. Let  $V$  and  $J$  be functions which together solve (3.1), so that  $J$  is a socially optimal carryout rule. Suppose that wherever  $J(x, x^a) = 0$  or  $K$  there exists some  $\epsilon > 0$  such that  $\gamma$  is constant along the line segment from  $(x - \epsilon, x^a)$  to  $(x + \epsilon, x^a)$ . (In words, suppose that whenever it is socially optimal for the private sector to stock out or to "stock up", taking policy as given, then the buffer authority has a zero "marginal propensity to store".) Then the function  $\psi$  given by  $\psi(s) = P(x - \gamma(s) - J(s)) \forall s$  is an equilibrium.

**Proof:** Suppose that  $\psi$  is not an equilibrium. Then one of two cases must hold. Under Case 1, for some  $s=(x, x^a)$ , we have  $J(s) < K$  and  $\psi(s) < \beta(1-\delta)E_h[\psi(s')]$ , where

$$x' = (1-\delta)(\gamma(s)+J(s)) + h', \quad (x^a)' = (1-\delta)\gamma(s), \quad \text{and } s'=(x', (x^a)').$$

Under Case 2, for some  $s$  we have  $J(s) > 0$  and  $\psi(s) > \beta(1-\delta)E_h[\psi(s')]$ .

Assume Case 1. Define  $W(\Delta) = U(x-\gamma(s)-J(s)-\Delta) + \beta E_h[U(x'+(1-\delta)\Delta-\gamma(x')-J(s'))]$ .

Now imagine a storage rule for the private sector that says "After period  $t+1$ , store the amount  $J(s')$  whenever the state vector is  $s'$ ", but in period  $t$  store  $J(s)+\Delta$  and in period  $t+1$  store an amount that leaves *total* storage what it would have been if  $\Delta$  was left equal to zero, i.e., store

$$J^* = J((1-\delta)(\gamma(s)+J(s))+h') - \gamma((1-\delta)(\gamma(s)+J(s)+\Delta)+h') + \\ \gamma((1-\delta)(\gamma(s)+J(s))+h').$$

If this rule is feasible, it will give utility values of  $W(\Delta)$  for the first two periods and then leave consumption, and hence utility, unchanged for all periods thereafter. Since  $W$  is differentiable with derivative equal to  $\beta(1-\delta)p_{t+1}-p_t > 0$ , a sufficiently small feasible value of  $\Delta > 0$  then represents a strict welfare improvement, thus contradicting the assumed optimality. The only task left is showing feasibility. In the case where

$$J(s') = J((1-\delta)(\gamma(s)+J(s))+h') > 0,$$

the continuity of  $\gamma$  ensures that for  $\Delta$  sufficiently small,  $J^*$  will be non-negative; the non-decreasing assumption on  $\gamma$  ensures that it will not be above  $K$  since  $J(s')$  is not; and  $J(s') \leq x' - \gamma(s')$  implies that

$$J^* = J(s') + \gamma(s') - \gamma((1-\delta)(\gamma(s)+J(s)+\Delta)+h') \leq x' - \gamma((1-\delta)(\gamma(s)+J(s)+\Delta)+h'),$$

which is the non-negative consumption constraint. Thus, in the case  $J(s') > 0$ , feasibility is assured for small enough  $\Delta$ . In the case  $J(s') = 0$ , by assumption for small enough  $\Delta$ , we have

$$\gamma((1-\delta)(\gamma(s)+J(s)+\Delta)+h') = \gamma((1-\delta)(\gamma(s)+J(s))+h'),$$

so that  $J^* = J(s')$ , which is already feasible. Thus, the non-optimality of  $J$  is proven, and thus we have a contradiction.

Case 2 is perfectly analogous, but with  $dW/d\Delta < 0$ . Thus, the appropriate  $\Delta$  is negative.

**Q.E.D.**

Thus, we can in some cases ensure the existence of a constrained efficient equilibrium. However, the conditions assumed in the Theorem, amounting to  $\gamma$  being flat enough of the time, are very strong and will not necessarily be satisfied. For example, suppose that  $\gamma$  is differentiable and nondecreasing in  $x$  and  $x^a$ . Also assume that  $k=0$  and " $K=\infty$ ". Consider the optimization problem:

$$V(s) = \max_{\{J\}} \left\{ U(x - J - \gamma(s)) + \beta E_h[V((1-\delta)(J + \gamma(s)) + h', (1-\delta)\gamma(s))] + \lambda J \right\}$$

where  $\lambda$  is the Kuhn-Tucker multiplier for the non-negativity constraint on private storage. The first order condition is:

$$p_t = \frac{dU}{dQ}(x - J - \gamma(s)) = \beta(1-\delta)E_h[V_1^+] + \lambda,$$

where subscripts indicate partial derivatives and a "plus" superscript on a function indicates that the function is evaluated at  $s' = ((1-\delta)(J + \gamma(s)) + h', (1-\delta)\gamma(s))$ . Of course,  $\lambda=0$  in the event that optimal storage is strictly positive. By the envelope theorem,  $V_1^+$  is equal to:

$$\begin{aligned} V_1^+ &= (1-\gamma_1) \frac{dU}{dQ}(x' - J' - \gamma(s')) + \beta(1-\delta)E_h[V_1^{++}\gamma_1^+ + V_2^{++}\gamma_1^+] \\ &= (1-\gamma_1)p_{t+1} + \beta(1-\delta)E_h[V_1^{++}\gamma_1^+ + V_2^{++}\gamma_1^+] \end{aligned}$$

where a double "plus" superscript on a function means it is evaluated at  $s'' = ((1-\delta)(J(s')+\gamma(s')), (1-\delta)\gamma)$  and  $h''$  is the (random) next-period update of  $h'$ . Putting this into the first order condition above and then using the first order condition for period  $t+1$  yields:

$$\begin{aligned} p_t &= \beta(1-\delta)E_{h'}[(1-\gamma_1^+)p_{t+1} + \beta(1-\delta)E_{h''}[V_1^{++}\gamma_1^+ + V_2^{++}\gamma_1^+]] \\ &= \beta(1-\delta)E_{h'}[p_{t+1} - \lambda\gamma_1^+ + \beta(1-\delta)E_{h''}[V_2^{++}\gamma_1^+]] \end{aligned}$$

in the event that optimal storage is strictly positive at time  $t$ . It is easy to show that if the buffer stock rule is non-decreasing in public stocks carried in, the value function is non-increasing in its second argument. Thus, if at time  $t$  optimal storage is positive (so that  $\lambda=0$ ) but there is a positive probability that it will be optimal at  $t+1$  for the private sector to carry no stocks (so that  $\lambda^+>0$ ) and that the buffer stock's marginal propensity to store will be positive, then the optimal program will not be an equilibrium. The optimum will have the feature that  $p^t < \beta(1-\delta)E_{h'}p_{t+1}$ , and thus under the optimal plan there will be an excess demand for speculative holdings.

This result is easy to interpret. If we hold policy to be exogenous, speculation imposes an externality on the rest of the market; each unit stored now will result in more grain sequestered by the government next period. This imposes a social cost because the buffer stock's storage is distortionary, but it is not a private cost to the speculator; thus the marginal unit of private storage involves zero private profit but a net loss in social welfare. It is likely that this reasoning would hold for the buffer stock rules in commonest use, for example, in price floor

schemes<sup>9</sup>.

This result is interesting for two reasons. First, it means that the technique of studying speculative equilibrium by first solving the social optimization problem, taking the environment as given, does *not* extend to markets subject to government intervention, even under perfect competition<sup>10</sup>. As Scheinkman and Schechtman (1983) demonstrated, that technique can produce a rich variety of results, but since most primary commodity markets have been subject to extensive intervention and a great many have long histories of buffer stocks, it seems that the alternative technique of studying fixed point properties of functional equations such as (2.2) will be more relevant in analyzing real world primary commodity markets.

Second, if we are to argue the case, studied by Newbery and Stiglitz (1981, Ch. 30), that a public buffer stock plan may be desirable for a given commodity, despite an absence of distortions in the market for that commodity, as a second best response to a distortion in another market (for example, a missing insurance market), then it is useful to know that the distortion introduced into the commodity market by a buffer scheme (i.e., too much storage given the optimization problem for that market in isolation) may be *magnified* by the response of rational speculators (i.e., they may store too much even given the excessive storage of the buffer

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<sup>9</sup>Under a price floor scheme, the buffer stock rule will have a unit marginal propensity to store at all states where  $x$  exceeds some critical value, and these states will be stock-out points which will be visited immediately after a period with private sector storage a positive fraction of the time (Williams and Wright, 1991, chapter 13). Suppose that the equilibrium is socially efficient given the stock rule. Then we get an immediate contradiction, because the equilibrium gives rise to a positive probability of a stock-out where the buffer stock has a positive marginal probability to store.

<sup>10</sup>An analogous argument was made by Kydland and Prescott (1977, footnote 7) in the context of the theory of investment.



authority) if they are beyond the Authority's control. Thus, if the problem is in some sense to equalize marginal distortions across markets, then accounting for the response of speculators will generally reduce the optimal scale of the buffer scheme.

An important qualification to this point, however, is that it holds only where the speculation is competitive. In the model of imperfectly competitive speculation of McLaren (1992a), it is shown that speculators store too *little* in the absence of intervention. It is easy to write down buffer stock rules for that model in which the speculators will still store too little given the buffer stock<sup>11</sup>.

#### **4. Policies that do not admit bubbles.**

So the existence of a constrained efficient equilibrium is in doubt. The natural next question is: does *any* equilibrium exist, and if so, is it unique?

A crucial concept in answering these questions is the regularity of the buffer stock function. The basic stated aim of real world buffer stocks is the stabilization of prices. Thus we are especially interested in the case in which  $\gamma$  is non-decreasing in  $x$ , or in other words, in which public sector stock demand is higher in periods of glut than in scarcity. We might think of such stock rules as "putatively stabilizing". However, if the public sector "marginal propensity to store" out of total availability was anywhere in excess of one, precisely if  $x - \gamma(x, x^a)$  failed

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<sup>11</sup>One example, if there are  $n$  speculators, is a rule in which the buffer stock behaves as if it was one of the speculators in an  $n+1$  member speculation oligopoly. In this case the equilibrium aggregate storage function would still lie below the competitive storage function. See McLaren (1992a) for details.

to be non-decreasing in  $x$ , then the stabilizing intent would be perverted because then in times of greater abundance less would be available to the private sector. We will call a buffer rule that has this property "over-eager". Finally, suppose that at the beginning of a given period a bag of grain fell from heaven and landed in the Authority's elevator. It would seem strange if the buffer stock manager responded by going out to buy more grain for storage than she was planning to buy before the manna drop. Precisely, it would seem strange if  $d - \gamma(x+d, x^a+d)$  failed to be non-decreasing in  $d$ . We will say that a rule  $\gamma$  is "regular" iff it is "putatively stabilizing" and not "over-eager" with respect to total availability and the Authority's own stocks, that is, iff  $\gamma$  is non-decreasing in  $x$  and in  $x^a$ ,  $(x - \gamma)$  is non-decreasing in  $x$ , and  $d - \gamma(x+d, x^a+d)$  is non-decreasing in  $d$ . This turns out to be crucial for existence and uniqueness of equilibrium.

With the imposition of this concept of regularity, it is fairly straightforward to prove the existence and uniqueness of a particular kind of equilibrium. This is an equilibrium characterized by three properties. First, it is non-increasing in  $x$ , so an increase in the total availability of grain will never cause the commodity's price to rise. Second, it is non-decreasing in  $x^a$ , so that an exogenous transfer of a bag from the private sector to the Authority's storage facility will not cause the price to fall. In a sense, the Authority can only make a difference by being more frugal than the private sector. Finally, we have  $\psi(x+d, x^a+d)$  non-increasing in  $d$ , so that a bag from the heavens that lands in the Authority's elevator will never cause a price rise. Call the class satisfying these properties  $\Psi$ . Then with a regular buffer stock there is a unique equilibrium in  $\Psi$ . All of this is stated formally and proven in Theorem 2, in the Appendix, which is perfectly parallel to the *laissez-faire* case treated by Deaton and Laroque (1992).

Thus, for the class of buffer stock rules which can reasonably be thought of as interesting,

existence is guaranteed. However, uniqueness is a much more difficult matter. In the next section, it will be shown that multiple equilibria are quite possible if  $\gamma$  is not regular; however, even if it is, all we have demonstrated is that any second equilibrium must lie outside of the class  $\Psi$ . This is analogous to the situation in Deaton and Laroque (1992), in which existence is proven only within the class of non-increasing equilibria. This seems to be a quite difficult problem. In the remainder of this section we will resolve it only for a special case -- the case in which public sector stock demand and equilibrium price are independent of the distribution of total availability between the public and private sectors.

We will show that in that case, any equilibrium price function must be non-increasing in  $x$ , and thus, must belong to  $\Psi$ . The proof will take two steps. First, we show that equilibrium carryout,  $x - P^{-1}(\psi(s))$ , must be non-decreasing for any equilibrium, and then we show how this leads to the monotonicity of the price function.

First, we need a lemma on the nature of total equilibrium carryout,  $x - P^{-1}(\psi(s))$ , where  $\psi$  is an equilibrium. The following demonstrates that if the public/private distribution of stocks does not matter, the carryout function can be flat only when speculators have hit a corner solution.

**Lemma 3.** Let  $\gamma$  be regular and let  $\psi$  be an equilibrium with  $\gamma$  on  $D_{\max}$ . Assume that  $\gamma$  and  $\psi$  do not depend on their second argument. If  $\tilde{x} > x$ ,  $\tilde{x}^a \geq x^a$ , and  $s = (x, x^a)$ ,  $\tilde{s} = (\tilde{x}, \tilde{x}^a)$ ,  $s, \tilde{s} \in D_{\max}$ , then:

$$(4.1) \quad x - P^{-1}(\psi(s)) = \tilde{x} - P^{-1}(\psi(\tilde{s}))$$

cannot hold unless both  $s$  and  $\tilde{s}$  are stock-out points or both are stocked-up points.

**Proof:** see Appendix.

This allows us to prove the monotonicity of equilibrium carryout.

**Theorem 3.** Let  $\gamma$  be regular and independent of  $x^a$ , and suppose  $\psi$  is an equilibrium which is also independent of  $x^a$ . Then  $x-P^{-1}(\psi(s))$  is non-decreasing in  $x$ .

**Proof:** Suppose that  $\tilde{x} > \bar{x}$  and  $\bar{x}-P^{-1}(\psi(\bar{x})) > \tilde{x}-P^{-1}(\psi(\tilde{x}))$ . Then by the Intermediate Value Theorem there exists  $\hat{x} \in (\bar{x}, \tilde{x})$  with

$$\bar{x}-P^{-1}(\psi(\bar{x})) > \hat{x}-P^{-1}(\psi(\hat{x})) > \tilde{x}-P^{-1}(\psi(\tilde{x})).$$

Similarly, since  $0 - \gamma(0, \cdot) = 0$ , there exists  $x \in (0, \bar{x})$  with

$$x-P^{-1}(\psi(x)) = \hat{x}-P^{-1}(\psi(\hat{x})).$$

Now, by Lemma 3, either  $s$  and  $\hat{s} = (\hat{x})$  are both stock-out points or they are both stocked-up points. In the first case, we have

$$\tilde{x}-P^{-1}(\psi(\tilde{x})) \geq \gamma(\tilde{x}) \geq \gamma(\hat{x}) = \hat{x}-P^{-1}(\psi(\hat{x})),$$

but that contradicts

$$\tilde{x}-P^{-1}(\psi(\tilde{x})) < \hat{x}-P^{-1}(\psi(\hat{x})).$$

In the second case, we have

$$\bar{x}-P^{-1}(\psi(\bar{x})) \leq \gamma(\bar{x}) + K \leq \gamma(\hat{x}) + K = \hat{x}-P^{-1}(\psi(\hat{x})),$$

but that contradicts

$$\bar{x}-P^{-1}(\psi(\bar{x})) > \hat{x}-P^{-1}(\psi(\hat{x})).$$

**Q.E.D.**

This leads quickly into a result on the monotonicity of the equilibrium price function.

**Theorem 4.** Let  $\gamma$  and  $\psi$  be as in Theorem 3. Then  $\psi$  is non-increasing in  $x$ .

**Proof:** Since  $\psi$  must satisfy  $\psi = T(\psi)$ , it is clear that it will also satisfy  $\psi = U(\psi)$ , where  $U$  is the functional operator given by:

$$U(\phi)(s) = \min \{ p(x - \gamma(s)), p(x - \gamma(s) - K), \\ \beta(1 - \delta)E_h[\phi((1 - \delta)(x - p^{-1}(\psi(s)) + h)) \} ]$$

$U$  is an operator on the class of continuous functions on  $D_{\max}$  whose range is a subset of the range of  $p$ , i.e., if  $\phi$  is a member of that class then so is  $U(\phi)$ . Now,  $U$  clearly has the properties that (i)  $\phi(s) \geq \phi'(s)$  for all  $s$  implies that  $U(\phi)(s) \geq U(\phi')(s)$  for all  $s$ ; (ii)  $U(\phi + a)(s) \leq U(\phi)(s) + \beta(1 - \delta)a$  for all  $s$  and for any constant  $a$ . Thus, again by Blackwell's lemma,  $U$  contracts with the supremum metric, and thus has a unique fixed point on the class of functions just indicated. Further, iterations of the form  $\phi_{i+1} = U(\phi_i)$ , beginning with any continuous function mapping into the range of  $p$ , will converge uniformly to that fixed point. Now, suppose that  $\phi_0$  is non-increasing in  $x$ . Then  $\hat{x} > x$  implies that  $U(\phi_0)(\hat{x}, x^a) \leq U(\phi_0)(x, x^a)$ , so that  $\phi_i$  is non-increasing in  $x$  for all  $i$ . Thus, the fixed point  $\psi$  must also have that property. **Q.E.D.**

An immediate consequence is that if  $\gamma$  is regular and does not depend on  $x^a$ , there is a unique equilibrium that does not depend on  $x^a$ . Further, any well-behaved restricted equilibrium must be identical to any other where they are both defined:

**Corollary 1.** Let  $\psi$  be an equilibrium with  $\gamma$  on  $D \subseteq D_{\max}$ , and let  $\gamma$  be regular and independent of  $x^a$ . Suppose that  $\psi$  can be extended to a function on  $D_{\max}$  that is a member of  $\Psi$ . Then if  $\phi$  is the unique equilibrium in  $\Psi$  on  $D_{\max}$  and if  $s \in D$ ,  $\psi(s) = \phi(s)$ .

**Proof:** Let  $\psi' \in \Psi$  be an extension of  $\psi$ . By Lemma 1,  $T(\psi') \in \Psi$  and by induction  $T^m(\psi') \in \Psi$  for all  $m \geq 1$ . Further, since  $\psi$  is an equilibrium on  $D$ ,  $s \in D$  implies that  $T(\psi')(s) = \psi'(s)$  and by induction  $T^m(\psi')(s) = \psi'(s)$  for all  $m \geq 1$ . But  $T^m(\psi')$  converges uniformly in  $m$  to  $\phi$ . Therefore,  $s \in D$  implies that  $\phi(s) = \psi'(s) = \psi(s)$ . **Q.E.D.**

Note that this is stronger than many uniqueness results in the commodities literature. Under laissez-faire, Scheinkman and Schechtman (1983) demonstrate uniqueness of a *socially optimal* equilibrium and Deaton and Laroque (1992) a *non-increasing* equilibrium. Here, within a certain class of buffer stock rules (which includes laissez-faire as a special case), we have shown uniqueness of an *equilibrium*. The only property we have had to impose is continuity.

The generalization to the case in which the distribution of total availability between the public and private sectors matters appears to be difficult. This begs the question: is it likely to matter in practice? It is plausible. The most obvious case in which the Authority's end-of-period stock demand may depend on its start-of-period holdings may be the case of credit constraints to which the Authority is subject. In this case it may conceivably wish in a given situation to stock 800,000 bags but only have 200,000 initially, and be able to find financing to buy only another 200,000; whereas if it had 700,000 bags initially, the available loan would be more than enough to meet its target. One can then imagine a credit-constrained buffer stock displaying a

"marginal propensity to store out of own stocks" of unity over a certain range. The analogy with "rule of thumb" households in applied work on consumption, who exhibit a unit marginal propensity to consume due to liquidity constraints is obvious. Further, in the real world, credit constraints for buffer schemes can indeed be important. The buffer stock scheme for tin essentially went bankrupt (Anderson and Gilbert, 1988).

Finally, we offer a limited policy neutrality result.

**Theorem 5.** Let  $K=\infty$ . Let  $\phi$  be the unique laissez faire equilibrium on  $D_{\max}$ , i.e.,

$$(4.2) \quad \phi(s) = \max\{P(x), \beta(1-\delta)E_h[\phi(s')]\} \quad \forall s \in D_{\max},$$

where  $s = (x)$  and  $s' = ((1-\delta)(x-P^{-1}(\phi(x))) + h')$ . Let  $\gamma$  be a regular buffer stock rule on  $D \subseteq D_{\max}$  which does not depend on  $x^a$ . Suppose that

$$(4.3) \quad x - P^{-1}(\phi(s)) \geq \gamma(s) \quad \forall s \in D.$$

Then  $\phi$  remains the unique equilibrium on  $D$  under buffer stock rule  $\gamma$ .

**Proof:** We wish to show that

$$(4.4) \quad \phi(s) = \max\{P(x-\gamma(s)), \beta(1-\delta)E_h[\phi(s')]\} \quad \forall s \in D.$$

First note that

$$P(x) > \beta(1-\delta)E_h[\phi(s')] \Rightarrow P(x-\gamma(s)) > \beta(1-\delta)E_h[\phi(s')].$$

Now, since  $\gamma(s) \leq x - P^{-1}(\phi(s)) \quad \forall s \in D$  by (4.3),  $P(x-\gamma(s)) \leq \phi(s) \quad \forall s \in D$ , so

$$P(x-\gamma(s)) > \beta(1-\delta)E_h[\phi(s')] \Rightarrow P(x) > \beta(1-\delta)E_h[\phi(s')] \text{ (by (4.2))}.$$

Thus,

$$P(x) > \beta(1-\delta)E_h[\phi(s')] \Leftrightarrow P(x-\gamma(s)) > \beta(1-\delta)E_h[\phi(s')].$$

In other words, the division of the state space into two regimes implied by the right hand side of (4.2) is exactly the same as that implied by the right hand side of (4.4). If, then, wherever the first regime prevails, that is, wherever  $\phi(s) = P(x)$ , we could show that  $P(x) = P(x-\gamma(s))$ , then the right hand sides of (4.2) and (4.4) would take the same value everywhere, and we would be done. Thus, note that

$$P(x) > \beta(1-\delta)E_h[\phi(s')] \Rightarrow \phi(x) = P(x) \Rightarrow x - P^{-1}(\phi(s)) = 0$$

$$\Rightarrow \gamma(x) = 0 \text{ (by (4.3))} \Rightarrow P(x) = P(x-\gamma(s)).$$

Thus, (4.4) must hold as required. The equilibrium is unique by theorems 2 and 4. **Q.E.D.**

Thus, if there is no capacity constraint which binds in equilibrium, a buffer stock will have *no* effect on prices unless for some value of the state vector the buffer stock is intended to carry more stocks than the private sector would in its absence (i.e., unless (4.3) is violated). This is analogous to a Modigliani-Miller theorem, since the result flows from the fact that under the stated conditions private agents can undo any meaningful effect of the buffer stock<sup>12</sup>. Unlike the Modigliani-Miller argument, here they can not restore the same asset holdings each trader would have held under *laissez faire*, since part of the available stock is locked in the buffer stock; however, once the effect on price is undone, they are indifferent as to the size of their holdings in the commodity and thus the resulting allocation is an equilibrium.

The theorem breaks down if either of two conditions holds. First, if there is a private sector capacity constraint which ever binds in equilibrium so that at some points in the state

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<sup>12</sup>I am grateful to Avinash Dixit for suggesting the analogy. For a survey of the basic issues surrounding the Modigliani-Miller theorem and a review of the literature see Gordon and Malkiel (1981).



space  $p_t < \beta(1-\delta)E_t p_{t+1} - k$ , then a buffer stock which does any storage at all at those points will lead to more total storage at those points and hence to a higher price<sup>13</sup>. This effect due to speculator capacity constraints is analogous to the failure of the Modigliani-Miller theorem when constraints are placed on the portfolios of investors, for example, borrowing constraints. Second, if the buffer stock ever holds more than traders would have held under *laissez faire*, obviously there must be more storage in equilibrium than would exist under *laissez faire*. This is analogous to the failure of Modigliani-Miller when investors cannot freely issue on their own account any security in the market; here the problem is that the traders cannot issue, say, their own coffee beans to replace those held by the buffer stock.

Note the empirical importance of this result. If the market is competitive and if private sector capacity constraints are unimportant, and if buffer stocks do have real effects, then they must in some state of the market store more than the private sector would in their absence. This would not plausibly be in states of large abundance (high  $x$ ), because those are the states in which the market would store the largest quantities of the good, and it is easy to document that buffer stocks in the real world generally have much smaller capacities than the private sector (for example, Hallwood, 1977). But that means that the buffer stocks must work by storing more than the private sector would in periods of relative *scarcity*, when the price is already high. This is not the way in which commodity price stabilization schemes are usually thought of. Some implications of this are drawn out in McLaren (1992b).

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<sup>13</sup> If it does not, then the traders must have reduced their storage at those points relative to what they would have stored under *laissez faire* by an amount equal to the buffer stock holdings. But then prices must be different anyway, because they must adjust to satisfy  $p_t \geq \beta(1-\delta)E_t p_{t+1} - k$  to make speculators willing to hold stocks below capacity.

Note that none of these paradoxes arises in the case of imperfectly competitive storage. In the model of McLaren (1992a), with  $n$  speculators, a buffer stock could enter, mimicking for example one speculator in an  $n+1$ -member equilibrium, and thus stabilize prices without ever storing more than the private sector would in its absence. It may thus be that observation of the way markets respond to buffer stock policies can tell us something about the underlying market structure.

## 5. Some policies which admit bubbles.

Suppose in a particular market there are two equilibria, one constrained efficient and the other not. Then we will adopt the convention of calling the latter equilibrium a "bubble".

Here we demonstrate the consequences of abandoning regularity of the buffer stock. Specifically, we offer an instance that generates a bubble. Since the result is somewhat surprising, we will demonstrate carefully that it really works.

Picture a market for a grain with a steady annual harvest of  $\bar{h}$ . For  $Q \in [(A-F), \infty)$ , with  $A-F < \bar{h}$ , consumer demand for the grain is given by  $P(Q) = Q^{-\alpha}$  where  $p$  is the price and  $Q$  is the quantity consumed in a given period. On the interval  $[0, (A-F)]$ ,  $P(Q)$  is just a straight-line extension of the curve so that the function is differentiable.  $\beta$  and  $\delta$  are as before.

A restricted equilibrium under laissez faire is obviously  $\phi(\bar{h}) = P(\bar{h})$ , with  $D = \{\bar{h}\}$ . This is unique (by Corollary 1) and is clearly also (constrained) efficient. With no possibility of drought, there is no social need or private incentive for the accumulation of stocks.

Now suppose that the government, suspicious of the ability of markets to govern

themselves, establishes a buffer stock authority to stabilize the market in case the greed of speculators should ever lead it astray. It charges the authority with the job of holding stocks in a glut and discharging all stocks in case of scarcity, following a rule illustrated in Figure 2. To make the rule simple, it defines a glut as a total availability (social carryin plus the harvest) of  $A'$  or more,  $\bar{h} < A' < A$ , and requires the stocks to rise linearly from 0 at  $A'$  to  $F$  at  $A$ ,  $F < A$ , and to remain at  $F$  for any glut more severe than  $A$ .

Now clearly  $\phi$  is still an equilibrium. If no one expects speculators to accumulate, there will be no expected change in price and thus no incentive to accumulate. The buffer stock rule will be irrelevant. However, if  $A$ ,  $A'$  and  $F$  are chosen right, there will also be a different and rather bizarre one, which we will call  $\psi$ . If there are no initial inventories but speculators believe that there will be some accumulated over periods 0 through  $T-1$ , say, at an accelerating rate, it is possible that the expected increasing shortage to consumers as speculators buy will cause the price to rise at exactly the rate  $[\beta(1-\delta)]^{-1}$ . In that case speculators will indeed be pleased to accumulate at the required rate. But this process cannot continue forever, and it will stop when stocks carried out reach a value of  $(A-\bar{h})/(1-\delta)$ , so that at time  $T$  total availability will be  $A$ , and the buffer authority will be obliged to take over the stock. The system then reaches a steady state, with carryout of  $F$  and total availability of  $(1-\delta)F + \bar{h}$  every period thereafter. Thus the buffer stock provides a vehicle for the ratification of the private sector's expectations. Let us examine a simple numerical example to make sure that this can really be an equilibrium.

Suppose that  $\bar{h} = 1$ ,  $\alpha = 2$ ,  $\delta=0.1$ ,  $\beta=0.9$ ,  $F=2$ ,  $A' = 2.2$ , and  $A = 2.4$ . Then consider the story told in Table I. Column (b) shows speculators' stock demand each period, (c) shows the quantity of stocks at the beginning of each period, (d) gives  $x_t$  each period, and (e) records

the price, which equals consumption raised to the power (-2). Consumption in turn is given by  $x_t$  minus the figure in (b) when  $x_t < 2.2$  and  $x_t$  minus (b) minus the public sector stock demand of 2 units when  $x_t > 2.4$ . The reader can verify that the price figure in the last column equals (to within rounding error) the availability figure in column (d) minus the stock demand in (b) minus 2 where appropriate, all raised to the power (-2).

TABLE I

(a)	(b)	(c)	(d)	(e)
t	Speculator end of period holdings	Carryin at t	Total availability at t ((c) plus harvest)	$p_t$
0	0.2420	0	1	1.7404
1	0.5356	0.2178	1.2178	2.1486
2	0.8680	0.4820	1.4820	2.6526
3	1.2286	0.7812	1.7812	3.2749
4	1.6084	1.1058	2.1058	4.0430
5	0	1.4476	2.4476	4.9914
6	0	1.8	2.8	1.5625
7	0	1.8	2.8	1.5625

Because prices rise at the rate  $[\beta(1-\delta)]^{-1}=(0.81)^{-1}$  from  $t=0$  to 5, speculators are willing to hold the quantities listed in (b). For example, in period 0, speculators are willing to hold 0.242 units, making the price  $(1-0.242)^{-2}=1.7404$ . Next period these stocks have been reduced by spoilage to 0.2178, at which time speculators buy even more of the harvest than last period, driving the price up to the anticipated level of  $1.7404/(0.81)^{-1}=2.1486$ . Through this the Authority

is inactive because supplies are still below the threshold, but at  $t=5$  total availability is  $2.4476 > A=2.4$ , so the Authority is obliged to buy  $F=2$  units. At this point speculators drop all stocks, realizing that prices next period will drop abruptly (in common parlance, "the bubble has burst"). Thereafter, stocks remain in the hands of government and at the level of  $F=2$  at the end of each period and  $(0.9)$  times this at the beginning of each period.

We can then define  $D'$  as the set of values of  $x_t$  listed in column (d), with  $\psi(x_t) = p_t$  as listed in column (e) for any  $x_t \in D'$ . It is elementary that  $D'$  satisfies "closure under  $\psi$ " and that  $\psi$  satisfies functional equation (2.2). By construction then,  $\psi$  satisfies the original definition of an equilibrium. Since  $D$  contains  $\bar{h}$  and  $\psi$  is certainly different from  $\phi$ , we have multiple equilibria at  $\bar{h}$ . Since  $\phi$  is constrained efficient but  $\psi$  clearly is not,  $\psi$  is a bubble.

The evolution of the two equilibria is plotted in Figure 3. Note that  $\psi$  is unambiguously worse in welfare terms. The price is higher at every moment than it is under  $\phi$ , and this means that at every moment consumption is lower.

We have thus discovered quite a surprising thing: a market with a unique, efficient equilibrium under laissez-faire, which develops a bubble when acted upon by a putatively stabilizing buffer stock intervention.

What drives this multiplicity of equilibria is the fact that there is a range in which the slope of  $\gamma$  exceeds unity. Thus, in the language of the preceding section, this is an example of the consequences of allowing "over-eagerness". Now we offer a sample of what may happen if the buffer stock rule is not putatively stabilizing. Let  $A$  and  $F$  be as above and let  $p_t, x_t, t=0, \dots, 5$ , and so on, be as in Table I. We modify  $\gamma$  slightly as shown in Figure 4. The new buffer stock rule rises from zero at  $A'$  to  $F$  at  $A$ , then falls from  $F$  at  $C'$  to zero at  $C$ ,

$x_4 < A' < A < C' < C < (1-\delta)F + \bar{h}$ , taking a value of zero for  $x \notin [A', C]$ . Perhaps we could think of this as a political equilibrium in which a Congress with a short memory is willing to support a buffer stock except in periods when very large accumulations demonstrate it to be a failure<sup>14</sup>. Note that this does nothing to the story we have told so far for  $t=0, \dots, 5$ , but how does it unfold from there? We will show that in this case we can choose  $C$  and  $C'$  such that there exists an equilibrium with the same behaviour as  $\psi$  for  $t=0$  to 5 but which then goes on to exhibit eternal, undamped deterministic cycles.

Suppose that  $C' = 2.77$  and  $C = 2.79$ . Then in period 6, when total availability reaches a value of 2.8, the Authority will stop holding grain. There will thus be a super-normal glut resulting from the collapse of the stabilization plan. One might then wonder if speculators would get back into the story. If, then, speculators purchase 1.95555 units, the price in period 6 will be 1.40235; next period will see stocks of  $0.9 \times 1.95555$  or 1.76 units, hence a total availability of 2.76. But this is within the range for reactivation of the buffer stock scheme. Thus, the Authority will again purchase 2 units, making the price 1.7313 provided speculators drop their stocks. This ratifies the decision of speculators in the previous period to purchase their 1.95555 units, since it implies a price rise of  $(0.81)^{-1}$ . Finally, since storage in period 7 equals 2 units, period 8 will be just like period 6; the price will fall to 1.40235, justifying the willingness of speculators to drop stocks at period 7. As before, this makes a formal equilibrium; we can add 2.76 to  $D'$  to define the domain  $D''$  and define  $\psi'(x) = \psi(x)$  for all points in  $D''$  with the exceptions that  $\psi'(2.8)=1.40235$  and  $\psi'(2.76)=1.7313$ . Trivially,  $D''$  is closed under  $\psi'$  and

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<sup>14</sup>We might think of this as an interpretation of U.S. grain policy. The enormous accumulations of the mid-80's resulted in political pressure to reduce government stocks drastically. See Babcock et. al., 1990, or *New York Times*, November 6, 1991, p. D1.

$\psi'$  satisfies (2.2) on  $D''$ .

Clearly, the price under equilibrium  $\psi'$  starting from  $x=\bar{h}$  traces an even stranger time path than it does under  $\psi$ . The price shoots up to 4.9914 as before, then rocks back and forth eternally between 1.40235 and 1.7313.

These results are interesting in the general theory of asset price bubbles because they do not require explosive price paths. Most models of bubbles require the price to be capable of growing exponentially for an indefinitely long period, which calls the empirical relevance of these models into question<sup>15</sup>. Here, multiple equilibria exist even with the price uniformly bounded above. This stands in parallel to the example by Obstfeld (1986) of a market for foreign exchange which will have one equilibrium under some monetary policies but multiple equilibria under others. In that model and in this one, it is not an explosive growth path that drives multiple asset price equilibria, but rather particular features of the way in which government is expected to respond to the market.

However, the empirical relevance of the particular examples suggested here is not clear. In the bubble equilibria shown here, the buffer stock authority must buy when the price is at its peak, which seems odd. McLaren (1992b) provides an example in which such behaviour is actually in the buffer stock manager's interest, because it sometimes has an interest in protecting speculators from losses.

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<sup>15</sup>Broze et. al. (1990, pp. 107-11) offer a model of commodity speculation with a continuum of equilibria indexed by a parameter  $\psi_0$ . When  $\psi_0 > 0$ , the price of the commodity grows explosively through time. Since they have a linear excess demand curve, this means that excess demand must grow explosively negative through time without end, and not only must inventories grow explosively but *additions* to inventories must grow explosively without limit. It is hard to justify this as a plausible real-world outcome.

## 6. Conclusion.

The main conclusions can be summarized thus:

- (i) Under a buffer stock scheme, perfectly competitive speculation will not necessarily deliver a socially optimal allocation of resources, but will under some conditions tend to magnify the distortion caused by the buffer stock. This is the opposite of the bias that has been found in some work on imperfectly competitive speculation.
- (ii) Buffer stock rules can be described as "regular" in a natural way involving various "marginal propensities to store" as being between zero and one. If these are satisfied, existence and uniqueness of competitive speculative equilibrium within an interesting class can be demonstrated, but full uniqueness appears to be a hard problem.
- (iii) A regular buffer stock will have no effect on anything if it does not store more than the whole private sector would store in its absence and private capacity constraints do not bind. This is another contrast with imperfectly competitive speculation, and has some surprising implications for interpreting actual buffer stocks of the past.
- (iv) If the buffer stock authority is not credit constrained and thus its behaviour does not depend on its inherited stocks, monotonicity of total storage and of price in total availability, and the uniqueness of equilibrium, are assured under perfect competition.
- (v) If the buffer stock does not satisfy the regularity conditions, weird multiple equilibria and what have been defined here as bubbles may exist. Thus, not only may a buffer stock plan fail to stabilize a market, but application of an "over-eager" buffer stock can *destabilize* a market by inducing a bubble where none was possible before.



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## Appendix: Existence and Uniqueness of one Kind of Equilibrium.

**Definition.** Define the functional operator  $T$  on functions  $g: D_{\max} \rightarrow \mathbb{R}$  by letting  $T(g)$  be the solution for  $f$  to:

$$(A.1) \quad f(s) = \text{middle} \{ (p(x - \gamma(s)), p(x - \gamma(s) - K), G(f(s), s) \}.$$

for all  $g$ . Clearly, from (2.2) a fixed point of  $T$  is an equilibrium. Now consider a function  $g: D_{\max} \rightarrow \mathbb{R}$ . Let

$$G(q, x, x^a) = \beta(1 - \delta)E_h[g((1 - \delta)(x - P^{-1}(q)) + h', (1 - \delta)\gamma(s)].$$

**Definition.** A buffer stock rule  $\gamma$  is *regular* if it is such that:

- (i)  $\gamma(x, x^a)$  and  $[x - \gamma(x, x^a)]$  are non-decreasing in  $x$ ;
- (ii)  $\gamma(x, x^a)$  is non-decreasing in  $x^a$ ; and
- (iii)  $d - \gamma(x + d, x^a + d)$  is non-decreasing in  $d$ .

**Lemma 1.** Let  $\gamma$  be regular. Assume that  $g(x + d, x^a + d)$  is continuous and is non-increasing in  $x$  and in  $d$  and non-decreasing in  $x^a$ . Then so is  $G(q, x + d, x^a + d)$ ; further,  $G$  is continuous and non-increasing in  $q$ .

**Proof:** First, note that any increase in  $x$  and  $x^a$  which does not move  $x^a$  faster than  $x$  will not increase  $g$ . That is, if  $\hat{x} \geq x$ ,  $\hat{x}^a \geq x^a$ , and  $(\hat{x} - x) \geq (\hat{x}^a - x^a)$ , then  $g(\hat{x}, \hat{x}^a) \leq g(x, x^a)$ . This is so because

$$g(x, x^a) \geq g(x + (\hat{x} - x), x^a + (\hat{x} - x)) = g(\hat{x}, x^a + (\hat{x} - x)) \geq g(x, \hat{x}^a).$$

Now let  $\hat{x} > x$  and  $\hat{x}^a > x^a$ .  $G(q, \hat{x}, x^a) \leq G(q, x, x^a)$  since

$$(1-\delta)(\hat{x}-x) \geq (1-\delta)[\gamma(\hat{x}, x^a) - \gamma(x, x^a)],$$

by regularity.  $G(q, x, \hat{x}^a) \geq G(q, x, x^a)$  since  $\gamma(x, \hat{x}^a) \geq \gamma(x, x^a)$ . And  $G(q, x+d, \hat{x}^a+d) \leq G(q, x, x^a)$ ,  $d>0$ , since

$$(1-\delta)d \geq (1-\delta)[\gamma(x+d, x^a+d) - \gamma(x, x^a)]$$

by regularity.

The monotonicity in  $q$  follows immediately from the fact that  $x-P^{-1}(q)$  is increasing in  $q$ .

**Q.E.D.**

**Lemma 2.** Let  $\gamma$  be regular. Assume that  $G$  satisfies the properties of Lemma 1.

Then:

(i) There exists a unique solution  $f(s)$  of (A.1) on  $D_{\max}$ . By definition,  $f=T(g)$ , as  $T$  is defined above. The function  $f$  is continuous and non-increasing in its first argument and non-decreasing in its second argument.

(ii)  $G_1(q,s) \geq G_2(q,s)$  for all  $(q,s)$  implies  $(T(G_1))(s) \geq (T(G_2))(s)$  for all  $(q,s)$ .

**Proof:** (i) For any state vector  $s$  there is a solution to (A.1) because setting  $f(s)=0$  makes it less than or equal to the right hand side and setting it equal to the maximum value of  $P$  makes it greater than or equal to the right hand side. Uniqueness follows from the fact that  $G$  is non-increasing in its first argument. Now consider a solution to (A.1) and a point  $s$  and raise the value of the  $x$  component of  $s$ . By the regularity of  $\gamma$ ,  $P(x-\gamma(s))$  and  $P(x-\gamma(s)-K)$  will not rise. Suppose that  $f(s)$  rose. Then, since  $G$  is non-increasing in its first and second arguments (from Lemma 1),  $G(f(s),s)$  will not rise. But then the left hand side of (A.1) has risen while the right

hand side has not, so the new point can not still be a solution. Thus, when  $x$  rises,  $f(s)$  must not rise. This establishes the non-increasing property of  $T(g)$  in  $x$ , and the non-decreasing property in  $x^a$  is established analogously.

(ii) Suppose  $G_1(q,s) \geq G_2(q,s)$  for all  $(q,s)$ . Suppose for some  $s$ ,  $T(G_1)(s) < T(G_2)(s)$ . Then since  $G_1$  is not increasing in its first argument,  $G_1(T(G_1)(s),s) \geq G_1(T(G_2)(s),s) \geq G_2(T(G_2)(s),s)$ , so the right hand side of (A.1) evaluated at  $s$  is no greater in the case of  $G_2$  than in the case of  $G_1$ . This contradicts the assumption that  $T(G_1)(s) < T(G_2)(s)$ . **Q.E.D.**

**Theorem 2.** Let  $\gamma : D_{\max} \rightarrow \mathbb{R}_+$  be regular. Define the class  $\Psi$  of continuous functions  $\psi : D_{\max} \rightarrow \text{range}(p)$  which have the property that  $\psi(x+d, x^a+d)$  is continuous and is non-increasing in  $x$  and non-decreasing in  $x^a$ , and is always non-increasing in  $d$ . Then there is a unique equilibrium in  $\Psi$ .

**Proof:** Recall that an equilibrium is a fixed point of the operator  $T$ . Lemma 1 shows that  $T$  maps elements of  $\Psi$  into  $\Psi$ . Define a metric  $\rho$  on  $\Psi$  by  $\rho(\psi, \phi) = \sup_{s \in D_{\max}} \{ |\psi(s) - \phi(s)| \}$ . It is elementary that  $T(\psi+a)(s) \leq \beta(1-\delta)a + T(\psi)(s)$  for all elements of  $\Psi$  and for any constant  $a$ . By Lemma 2,  $\psi(s) \geq \phi(s)$  for all  $s \in D_{\max}$  implies that  $T(\psi)(s) \geq T(\phi)(s)$  for all  $s \in D_{\max}$ . Thus, Blackwell's Lemma applies, and  $T$  is a contraction on  $\Psi$ . Existence and uniqueness follow immediately. **Q.E.D.**

### Proof of Lemma on Equilibrium Carryout.

**Lemma 3.** Let  $\gamma$  be regular and let  $\psi$  be an equilibrium with  $\gamma$  on  $D_{\max}$ . Assume that  $\gamma$  and  $\psi$  do not depend on their second argument. If  $\tilde{x} > x$ ,  $\tilde{x}^a \geq x^a$ , and  $s = (x, x^a)$ ,  $\tilde{s} = (\tilde{x}, \tilde{x}^a)$ ,  $s, \tilde{s} \in D_{\max}$ , then:

$$(4.1) \quad x - P^{-1}(\psi(s)) = \tilde{x} - P^{-1}(\psi(\tilde{s}))$$

cannot hold unless both  $s$  and  $\tilde{s}$  are stock-out points or both are stocked-up points.

**Proof:** Suppose that (4.1) holds. First note that  $\tilde{x} > x$  and (4.1) imply that  $\psi(\tilde{s}) < \psi(s)$ . Further, note that if  $s'$  and  $\tilde{s}'$  are the respective next period state vectors as defined above, (4.1) implies that  $\beta(1-\delta)E_h[\psi(s')] = \beta(1-\delta)E_h[\psi(\tilde{s}')]$ . Next, we break the problem into four cases.

Case 1.  $\psi(s) = \beta(1-\delta)E_h[\psi(s')]$ . In this case, assume that

$$\psi(\tilde{s}) \geq \beta(1-\delta)E_h[\psi(\tilde{s})].$$

Then  $\psi(s) = \beta(1-\delta)E_h[\psi(s')] = \beta(1-\delta)E_h[\psi(\tilde{s}')] \leq \psi(\tilde{s})$ . But  $\psi(s) > \psi(\tilde{s})$ . This is a contradiction. Therefore,  $\psi(\tilde{s}) < \beta(1-\delta)E_h[\psi(\tilde{s})]$ , so  $\psi(\tilde{s}) = P(x - \gamma(\tilde{s}) - K)$  (i.e.,  $\tilde{s}$  is a stocked-up point). But now by (4.1),

$$\gamma(s) \leq x - P^{-1}(\psi(s)) = x - P^{-1}(\psi(\tilde{s})) = \gamma(\tilde{s}) + K.$$

Thus,  $x - P^{-1}(\psi(s)) - \gamma(s) = \gamma(\tilde{s}) - \gamma(s) + K \geq K$ , but  $x - P^{-1}(\psi(s)) - \gamma(s) \leq K$  because of the capacity constraint. Thus,  $\psi(s) = P(x - \gamma(s) - K)$ , and  $s$  is a stocked-up point also.

Case 2.  $\psi(\tilde{s}) = \beta(1-\delta)E_h[\psi(\tilde{s}')]$ . Assume that  $\psi(s) \leq \beta(1-\delta)E_h[\psi(s')]$ . Then we would have

$$\psi(\tilde{s}) = \beta(1-\delta)E_h[\psi(\tilde{s}')] = \beta(1-\delta)E_h[\psi(s')] \geq \psi(s).$$

But then  $\psi(\tilde{s}) \geq \psi(s)$ . This is a contradiction, so  $s$  must be a stock-out point. Consider the two

possibilities allowed by the regularity of  $\gamma$ , namely,  $\gamma(s) = \gamma(\tilde{s})$  or  $\gamma(s) < \gamma(\tilde{s})$ . First, let  $\gamma(s) = \gamma(\tilde{s})$ . Then

$$\tilde{x} - P^{-1}(\psi(\tilde{s})) - \gamma(\tilde{s}) = x - P^{-1}(\psi(s)) - \gamma(s) = 0,$$

so  $\psi(\tilde{s}) = P(\tilde{x} - \gamma(\tilde{s}))$ , or  $\tilde{s}$  is a stock-out point as well. Now let  $\gamma(s) < \gamma(\tilde{s})$ . Then

$$\tilde{x} - P^{-1}(\psi(\tilde{s})) - \gamma(\tilde{s}) < x - P^{-1}(\psi(s)) - \gamma(s) = 0, \text{ so } \psi(\tilde{s}) < P(\tilde{x} - \gamma(\tilde{s})) < P(\tilde{x} - \gamma(\tilde{s}) - K).$$

But this violates the requirement that

$$\psi(s) = \text{middle} \{ P(x - \gamma(\tilde{s})), P(x - \gamma(\tilde{s}) - K), \beta(1 - \delta)E_h[\psi(\tilde{s}')] \}.$$

Case 3.  $\tilde{s}$  is a stock-out point and  $s$  is a stocked-up point. In other words,  $\psi(\tilde{s}) = P(\tilde{x} - \gamma(\tilde{s}))$  and  $\psi(s) = P(x - \gamma(s) - K)$ . This cannot occur. If it did, we would have

$$\psi(\tilde{s}) = P(\tilde{x} - \gamma(\tilde{s})) \geq \beta(1 - \delta)E_h[\psi(\tilde{s}')] = \beta(1 - \delta)E_h[\psi(s')] \geq P(x - \gamma(s) - K) = \psi(s)$$

(by the functional equation (2.2)). Thus,  $\psi(\tilde{s}) \geq \psi(s)$ , a contradiction.

Case 4.  $\psi(\tilde{s}) = P(\tilde{x} - \gamma(\tilde{s}) - K)$ , and  $\psi(s) = P(x - \gamma(s))$ . This cannot occur, since, by (4.1), it implies that  $\gamma(\tilde{s}) + K = \gamma(s)$ . But the left hand side of this is strictly greater than the right hand side.

Thus, each of these four cases is either impossible or implies that  $s$  and  $\tilde{s}$  are either both stock-out points or both stocked-up points. Since these are the only two possibilities not covered by cases 1 through 4, the lemma is proved. **Q.E.D.**

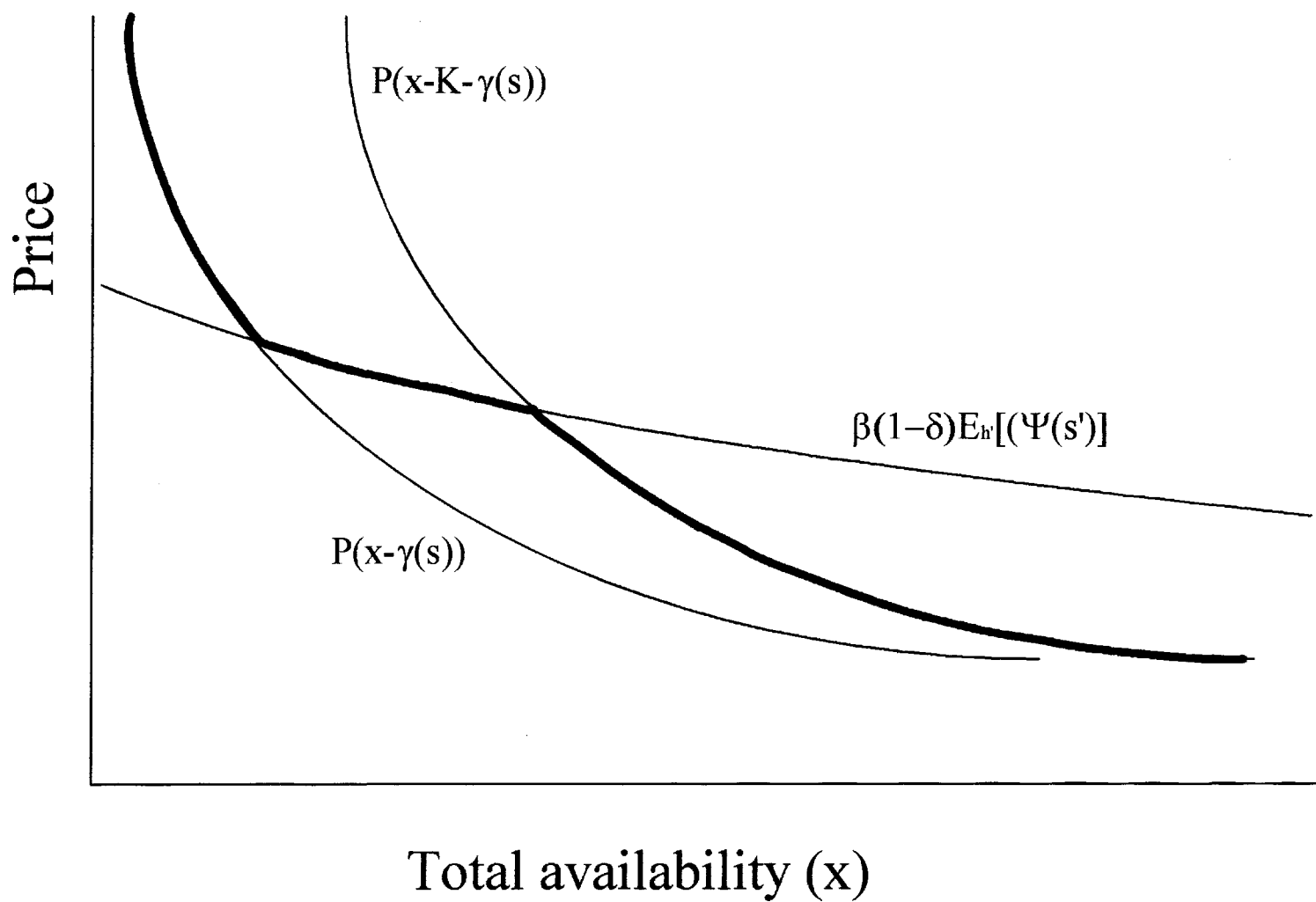


Figure 1: The construction of equilibrium.



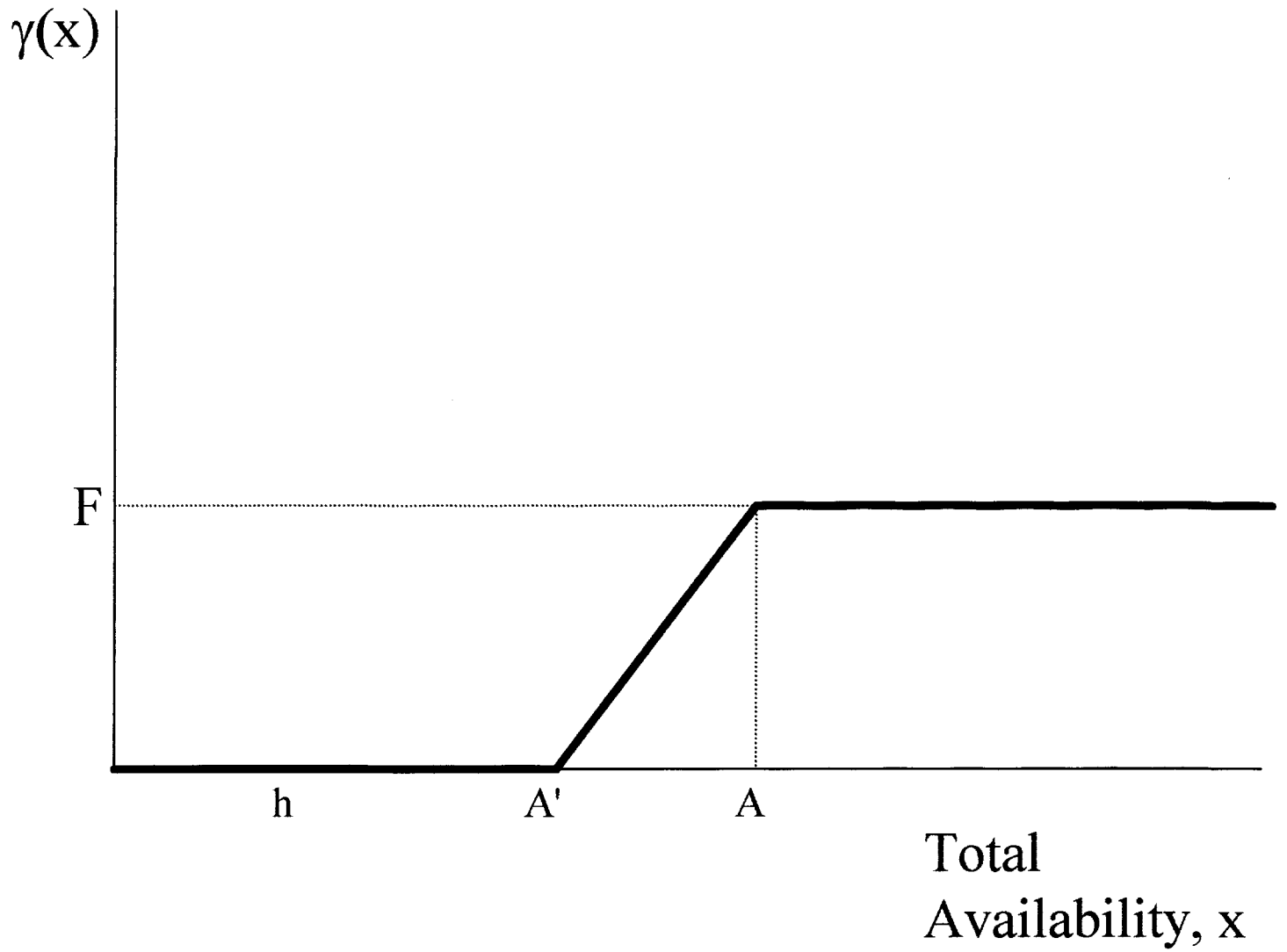


Figure 2: A buffer stock rule

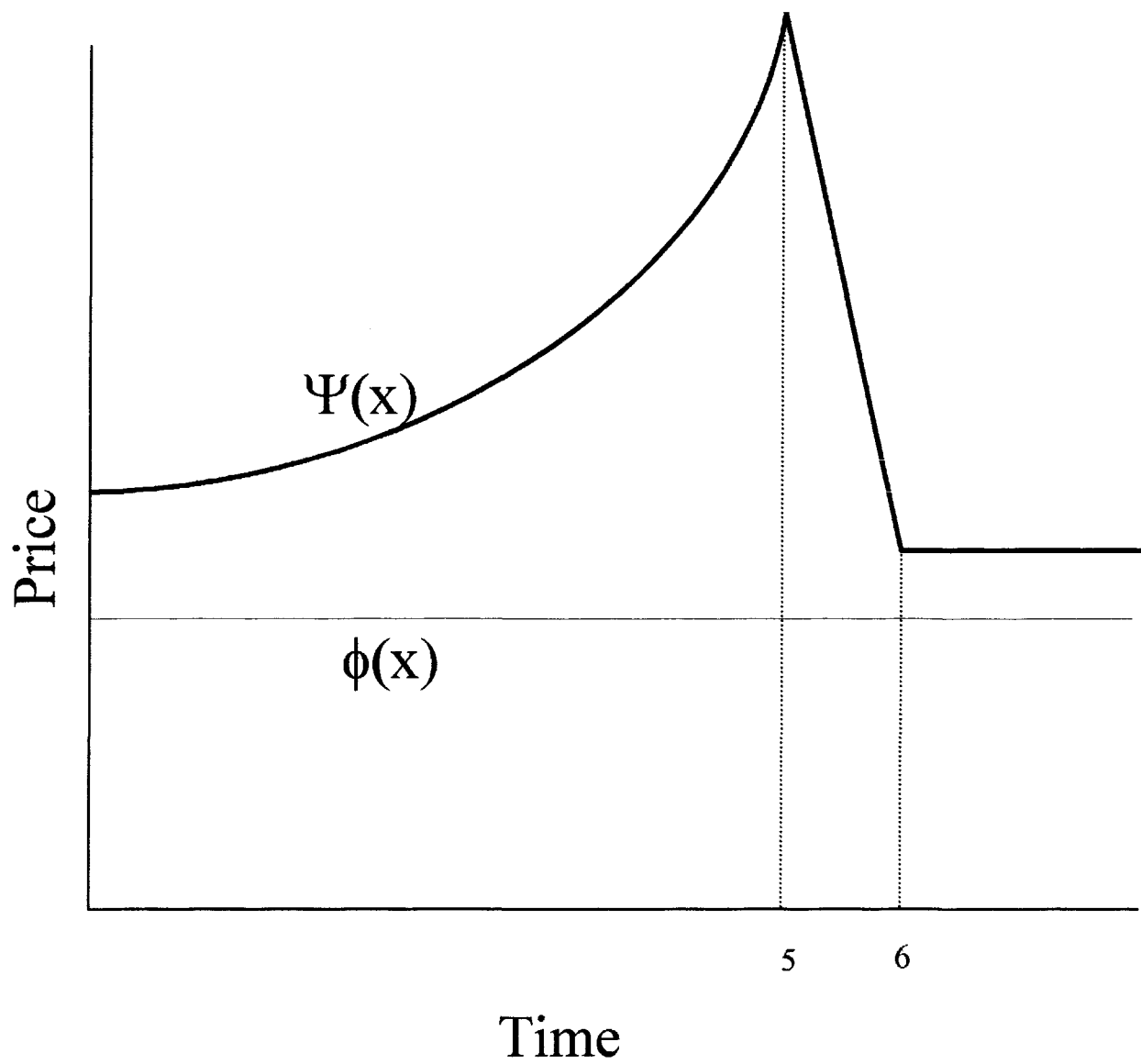


Figure 3: The progress of equilibrium

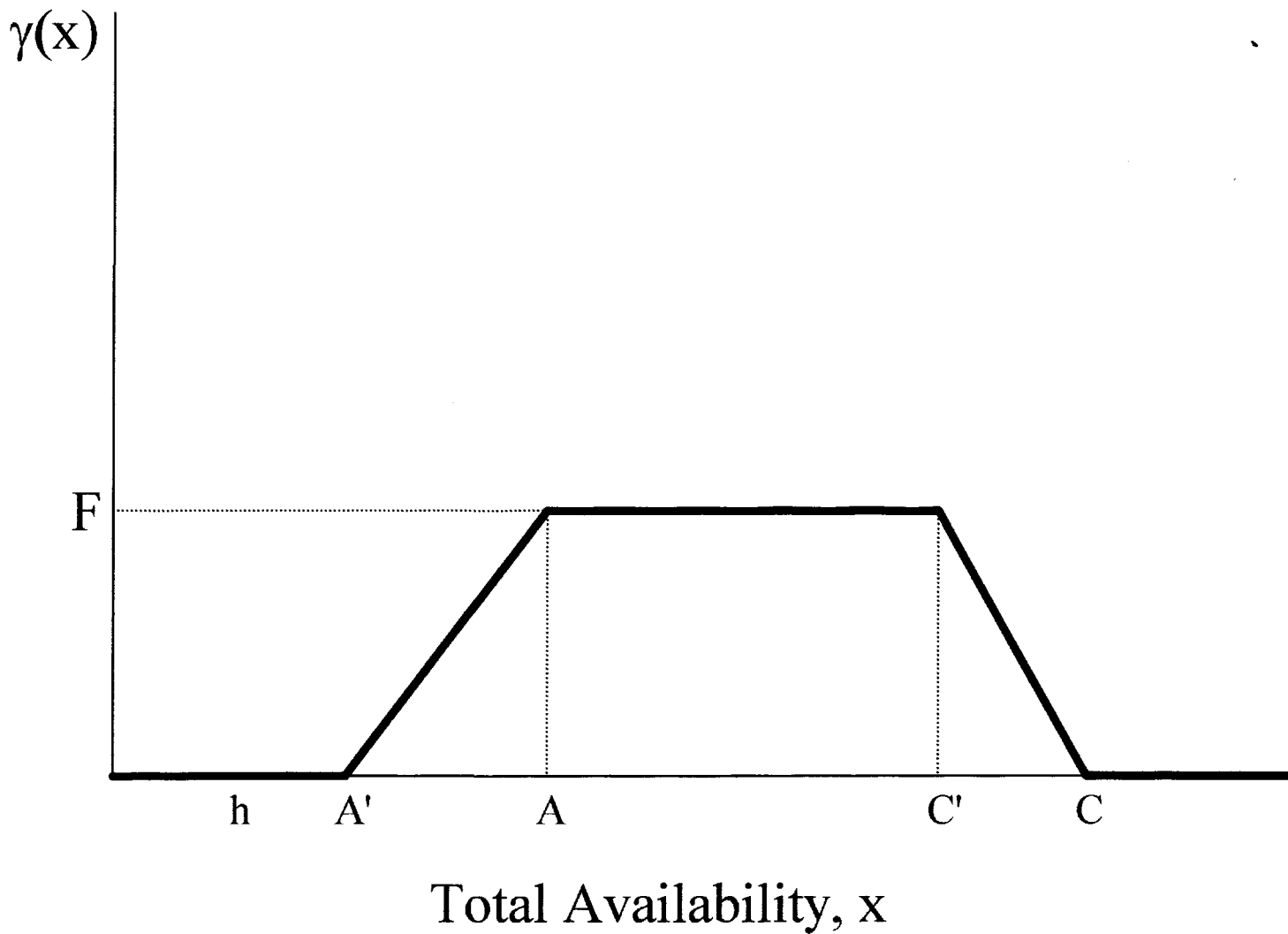


Figure 4: A buffer stock rule which is not putatively stabilizing